Calculus Test (Part I)

(Problem 1)

$$\lim_{x\to -3} \frac{x^2-9}{x+3} = \frac{(x-3)(x+3)}{(x+3)} = x-3 = \lim_{x\to -3} = -3-3 = \boxed{-6}$$

(Problem 2)

$$\lim_{x\to 3} \frac{\frac{x}{x+2} - \frac{3}{5}}{x-3} = \frac{\frac{5(x)}{5(x+2)} - \frac{3(x+2)}{5(x+2)}}{x-3} = \frac{\frac{5x-3x-6}{5(x+2)}}{x-3} = \frac{\frac{2x-6}{5(x+2)}}{x-3} = \frac{2(x-3)}{5(x+2)} \cdot \frac{1}{(x-3)}$$

$$\lim_{x\to 3} \frac{2}{5(x+2)} = \frac{2}{5(3+2)} = \boxed{\frac{2}{25}}$$

(Problem 3)

$$\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - [x^2 - 2x + 1]}{\Delta x}$$

$$\lim_{\Delta x\to 0} \frac{x^2 + 2\Delta x x + \Delta x^2 - 2x - 2\Delta x + 1 - x^2 + 2x - 1}{\Delta x}$$

$$\lim_{\Delta x\to 0} \frac{2\Delta x \cdot x + \Delta x^2 - 2\Delta x}{\Delta x}$$

$$\lim_{\Delta x\to 0} 2x + \Delta x - 2 = \boxed{2x-2}$$

(Problem 4)

$$\lim_{x\to 0} \frac{x}{\sin 3x} = \lim_{x\to 0} \frac{1}{3} \cdot \frac{3x}{\sin 3x} = \lim_{x\to 0} \frac{1}{3} \cdot \lim_{x\to 0} \frac{3x}{\sin 3x} = \boxed{\frac{1}{3}}$$

(Problem 5)

$$\lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x-3} = \lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2} = \frac{x+1-4}{(x-3)(\sqrt{x+1} + 2)}$$

$$\lim_{x\to 3} \frac{(x-3)}{(x-3)(\sqrt{x+1} + 2)} = \lim_{x\to 3} \frac{1}{\sqrt{x+1} + 2} = \frac{1}{\sqrt{3+1} + 2} = \boxed{\frac{1}{4}}$$

(PROBLEM 6)

$$\lim_{\theta \to 0} \frac{\sec\theta - 1}{\theta \sec\theta} = \lim_{\theta \to 0} \frac{\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{\theta \frac{1}{\cos\theta}} = \lim_{\theta \to 0} \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\theta}{\cos\theta}}$$

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{\cos\theta} \cdot \frac{\cos\theta}{\theta} = \lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} = \boxed{0}$$

(PROBLEM 7)

$$\lim_{x \to 0} \frac{\sin 4x}{5x} = \lim_{x \to 0} \frac{4}{5} \cdot \frac{\sin 4x}{\frac{4}{5} \cdot 5x}$$

$$\lim_{x \to 0} \frac{4}{5} \frac{\sin 4x}{4x} = \boxed{\frac{4}{5}}$$

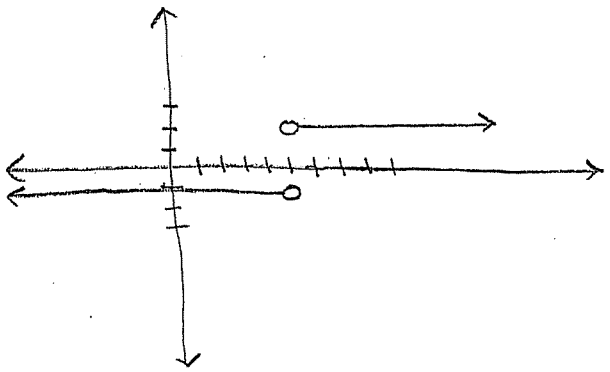
(PROBLEM 8)

$$\lim_{x \to 0^+} \left(x - \frac{1}{x^3} \right)$$

$$\lim_{x \to 0^+} \quad ,00001 - \frac{1}{(,00001)^3} = -\infty$$

(PROBLEM 9)

GRAPH $f(x) = \dfrac{|x-5|}{x-5}$



DISCONTINUOUS AT $x = 5$

a) $\lim_{x \to 5^+} f(x) = 1$

b) $\lim_{x \to 5^-} f(x) = -1$

c) $\lim_{x \to 5} f(x)$: DOES NOT EXIST

(PROBLEM 10)

GRAPH $f(x) = \begin{cases} \dfrac{x^2 - 5x - 14}{x+2}, & x \leq 7 \\ \sqrt{x-7} + 1, & x > 7 \end{cases}$

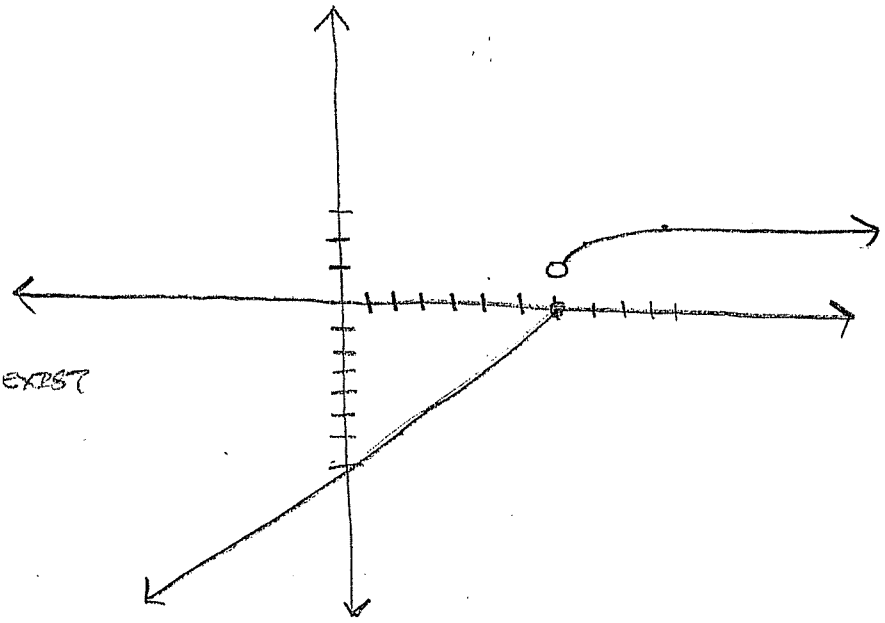
$\dfrac{(x-7)(x+2)}{x+2}$

DISCONTINUOUS AT $x = 7$

a) $\lim_{x \to 7^+} f(x) = 1$

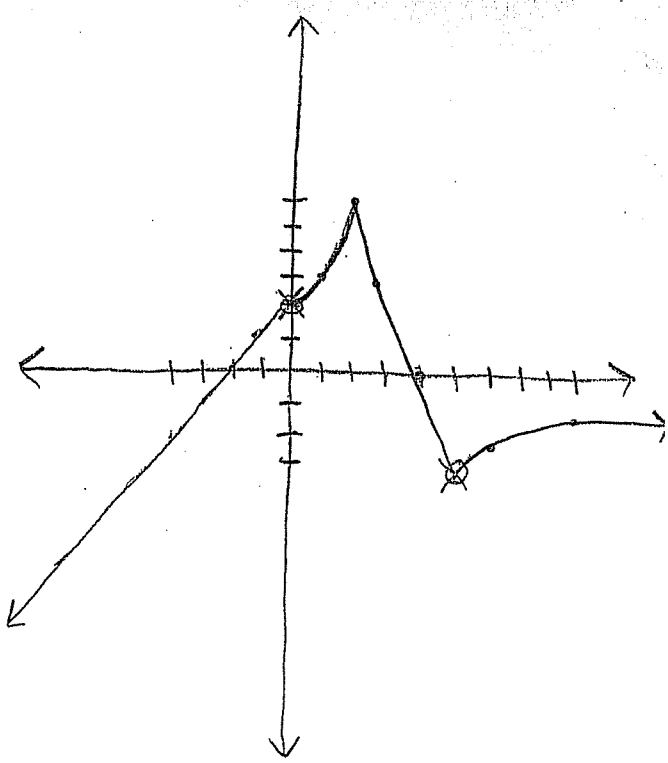
b) $\lim_{x \to 7^-} f(x) = 0$

c) $\lim_{x \to 7} f(x)$ = DOES NOT EXIST



(PROBLEM 11)

GRAPH $f(x) = \begin{cases} x^2 + 2, & 0 \leq x \leq 2 \\ x + 2, & x < 0 \\ -3x + 12, & 2 < x < 5 \\ \sqrt{x-5} - 3, & x \geq 5 \end{cases}$



a) $\lim_{x \to 0} f(x) = 2$

b) $\lim_{x \to 2} f(x) = 6$

c) $\lim_{x \to 5} f(x) = -3$

D) $\lim_{x \to -5} f(x) = -3$

E) $\lim_{x \to 9} f(x) = -1$

F) $\lim_{x \to 4} f(x) = 0$